IACS Unified Requirements for Polar Ships

Background Notes to

Design Ice Loads

Prepared for:

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Summary

This document describes how the design ice loads in the IACS Unified Requirements for Polar Ships have been developed. The proposed UR is based on the concept that ice loads can be rationally linked to the design scenario. The design scenario is a glancing collision with an ice edge (edge of a channel, edge of a floe). The ice load model assumes a 'Popov' [5] type of collision, with ice indentation described by a pressure-area relationship (see ref [1]). The normal ice load, expressed in its complete form (see a13 and a17 from Annex A) is

$$F_n = (3+2\cdot ex)^{\frac{2+2\cdot ex}{3+2\cdot ex}} \cdot Po^{\frac{1}{3+2\cdot ex}} \cdot \left(\frac{\tan(\phi/2)}{\sin(\beta')\cdot\cos^2(\beta')}\right)^{\frac{1+ex}{3+2\cdot ex}} \cdot \left(\frac{1}{2}\Delta_n \cdot V_n^2\right)^{\frac{2+2\cdot ex}{3+2\cdot ex}} \qquad \text{(s.1)}$$

This equation can be expressed in simpler terms, as a design equation. With ex =-0.1, ϕ = 150 deg, and all hull angle terms collected into a term called *fa*, the equation becomes

$$F_n = fa \cdot Po^{.36} \cdot \Delta_{ship}^{.64} \cdot V_{ship}^{1.28} \qquad \text{(s.2)}$$

where fa contains the hull angle effects,

$$fa = \left(0.097 - .68 \cdot \left(\frac{x}{L} - .15\right)^2\right)\frac{\alpha}{\sqrt{\beta'}} \qquad \text{(s.3)}$$

Equation (s.2) is further simplified by converting class-dependant ship and ice parameters into class factors, as

$$F_n = fa \cdot CF_C \cdot \Delta_{ship}^{.64} \qquad \text{\{bow region\}} \qquad \text{(s.4)}$$

where

CF_C = Crushing Class Factor = $Po^{0.36}\ \ V_{ship}{}^{1.28}$

Similar equations are derived for line load Q and patch pressure p. The full derivations of the equations are shown in Annex A.

1. Introduction

This document describes the principles upon which the design ice loads in the IACS Unified Requirements for Polar Ships have been developed. Also included is a discussion on ship and ice parameters as well as background details on the methodology used to calculate the design load.

The proposed UR is based on the concept that ice loads can be rationally linked to the design scenario. The design scenario is a glancing collision with an ice edge (edge of a channel, edge of a floe). The form of the load equation is derived from the solution of an energy based collision model in which the kinetic energy is equated to ice crushing energy. Ice thickness, ice strength (crushing pressures), hull form, ship size and ship speed are all taken into account. The results are in close agreement with and are supported by a variety of studies, including numerical models, model tests, ship trials and operational experience.

The forces generated during a glancing impact are represented in ways that allow them to be used in developing scantlings for individual structural elements, grillages, and supporting structure. Impact loads on the forebody of the ship are converted to loads on other hull areas by an area factor. This approach is like those in current ice class rules.

The full derivations of many of the UR equations are quite lengthy but are presented in Annexes to this document. This document has a number of references, many of which were produced during the UR development process. The referenced documents can provide more details of the rationale behind the selection of the methods and assumptions presented here.

2. Design scenario and development of design loads

The design scenario that forms the basis of the ice loads for plating and framing design is a glancing collision on the shoulders of the bow (see Figure 1, 2). In this scenario, the ship is assumed to be moving forward at the design speed, striking an angular ice edge. During the collision, the ship penetrates the ice and rebounds away. The ship speed, ice thickness and ice strength are assumed to be class dependent. The maximum force can be found by equating the normal kinetic energy with the energy used to crush the ice. The ice crushing force cannot exceed the force required to fail the ice in bending. The combination of angles, ice strength and thickness determine the force limit due to bending.

The rule scenario is strictly valid only for the bow region, and for the stern of double-acting ships. In order to produce a balanced structural design, loads on other hull areas are set as a proportion of the bow area by using empirical hull area factors as described in ref [1]. The loads on other hull areas are not strongly dependent on bow angles, and so bow loads are normalized using a 'standard' set of bow angles before being applied elsewhere..

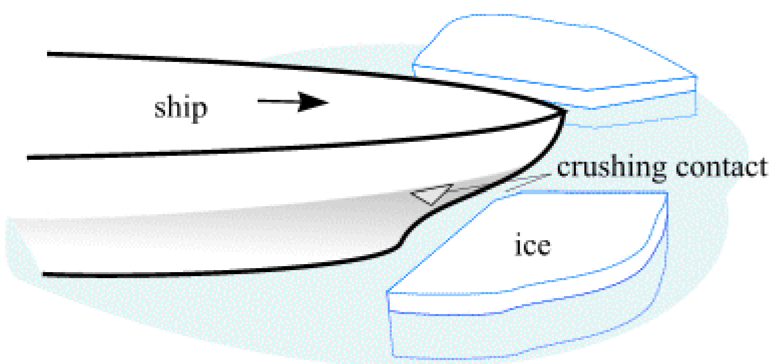

Figure 1. Design scenario - glancing collision on shoulder.

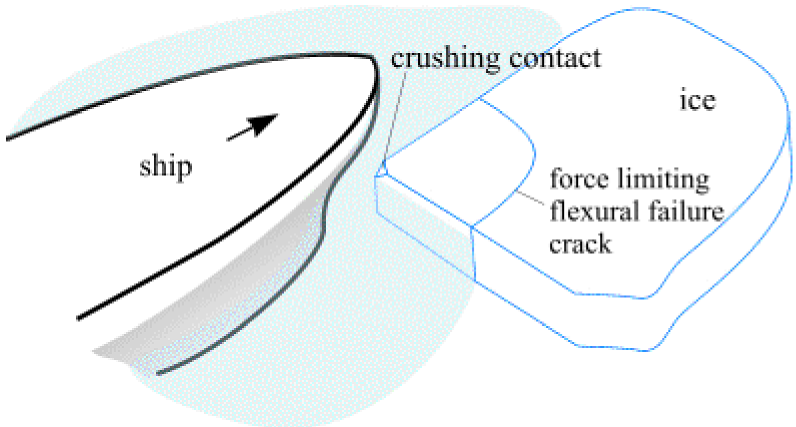

Figure 2. Design scenario - flexural failure during glancing collision.

The design loads are developed in several stages. First, the total load is found as the minimum of the crushing and flexural limiting loads for the design ice. Second, the patch over which this load is applied is determined and idealized. Third, the distribution of load within the patch is modified to account for local loading peaks. Each stage is described in the following sections.

3. *Ice crushing force derivation*

The ice load is derived for an oblique collision on the bow. The ice load model assumes a 'Popov' type of collision, with ice indentation described by a pressure-area relationship (see ref [1]). The normal ice load, expressed in its complete form (see a13 and a17 from Annex A) is

$$F_n = (3+2\cdot ex)^{\frac{2+2\cdot ex}{3+2\cdot ex}} \cdot Po^{\frac{1}{3+2\cdot ex}} \cdot \left(\frac{\tan(\phi/2)}{\sin(\beta')\cdot\cos^2(\beta')}\right)^{\frac{1+ex}{3+2\cdot ex}} \cdot \left(\frac{1}{2}\Delta_n \cdot V_n^2\right)^{\frac{2+2\cdot ex}{3+2\cdot ex}} \qquad (1)$$

where

Po = ice pressure (at 1 m^2) [Mpa] <class dependent>
ex = pressure-area exponent [no units, assume ex = -0.1]
ϕ = ice edge opening angle [assume 150 deg]
β' = normal frame angle (see Figure 3, 4)
Δ_n = normalized mass (= Δ_{ship}/Co)
V_n = normalized velocity (= V_{ship} l) < V_{ship} is class dependent>
Co = mass reduction coefficient
l = x-direction cosine (l = sin(α) cos(β'))

This same equation can be expressed in simpler terms, as a design equation. With ex = - 0.1, ϕ = 150 deg, and all hull angle terms collected into a term called *fa*, the equation becomes

$$F_n = fa \cdot Po^{.36} \cdot \Delta_{ship}^{.64} \cdot V_{ship}^{1.28} \qquad (2)$$

where

$$fa = 1.94 \cdot \left(\frac{\tan(\phi/2)}{\sin(\beta')\cdot\cos^2(\beta')}\right)^{.32} \cdot \left(\frac{1}{2\cdot Co}\cdot l^2\right)^{.64} \qquad (3)$$

Equation (3) is quite complex for a rule equation (mainly because Co is quite complex – see Annex B). The following simplified equation is proposed instead,

$$fa = \left(0.097 - .68\cdot\left(\frac{x}{L} - .15\right)^2\right)\frac{\alpha}{\sqrt{\beta'}} \qquad (4)$$

To validate equation (4), a number of different hull forms (see Figures 5 and 6) were examined. Figure 7 shows that the comparison is very good over most of the possible range of angles. The simplified equation is limited to have a maximum value of 0.6 to avoid generating extreme values.

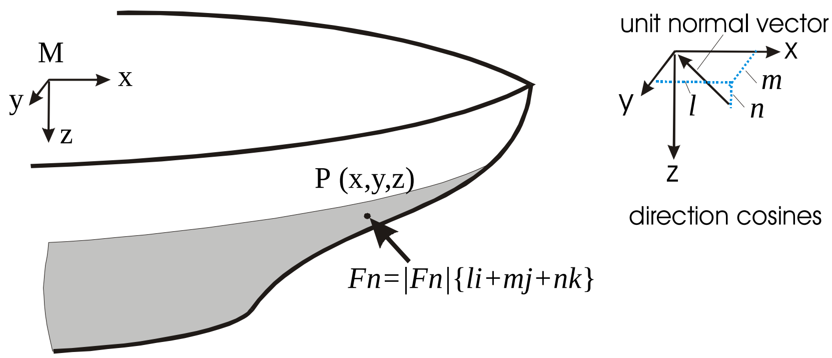

Figure 3. Collision geometry

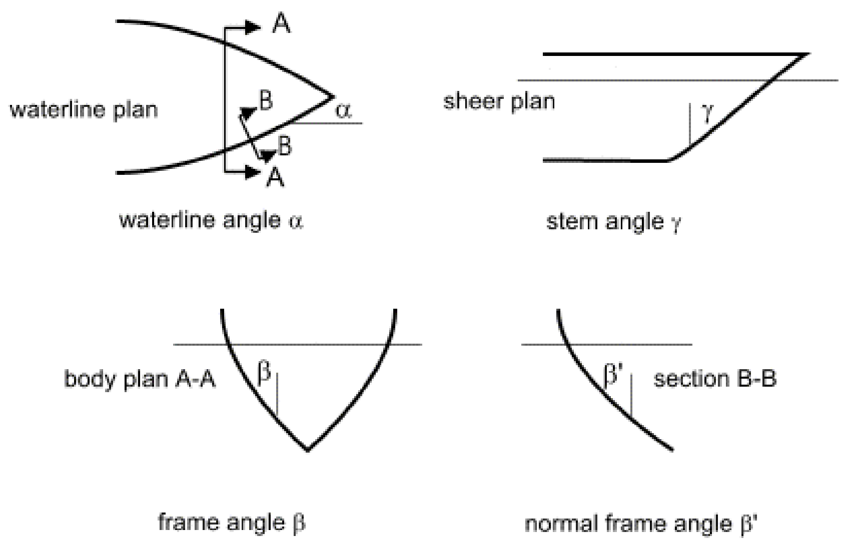

Figure 4. Definition of hull angles.

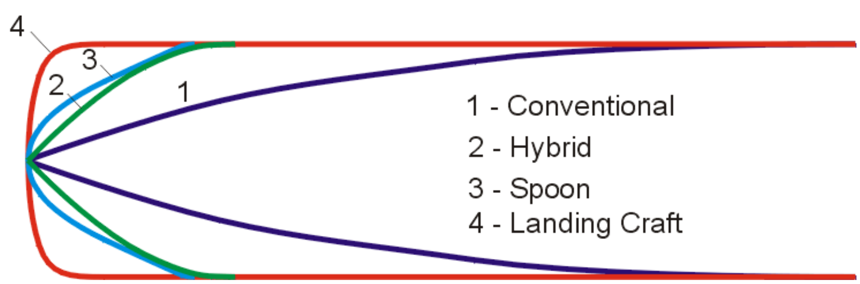

Figure 5. Waterlines of 4 hull families examined.

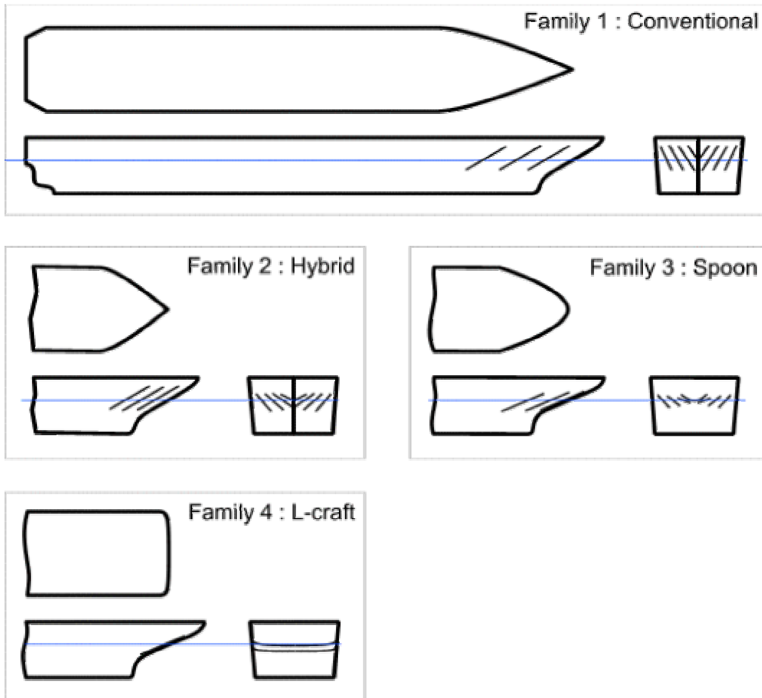

Figure 6. Hull forms of 4 families examined.

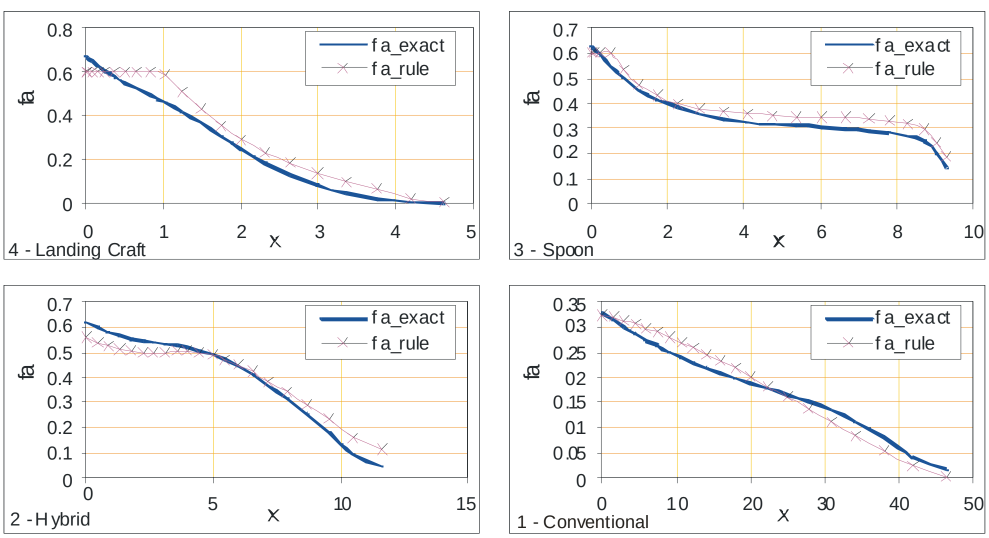

Figure 7. Comparison of exact (eqn 3) and rule (eqn 4) functions for the angle factor *fa*.

4. Ice flexural failure

The above equations only consider the crushing interaction. If there is flexural failure, the ice force will be limited. There are different angle influences in the crushing and flexural forces. The maximum force depends on the combination of effects. Furthermore, it is impossible to express all the angle influences in one precise equation.

The normal force is limited to

$$F_{n,\lim} = \frac{1}{\sin(\beta')} \cdot 1.2 \cdot \sigma_f \cdot h_{ice}^2 \qquad (5)$$

where

h_{ice} = ice thickness [m] <class dependent>

σ_f = ice flexural strength [Mpa] <class dependent>

β' = normal (true) frame angle

This is a variant of the standard ice flexural strength equation, with the downward force component matched to the 'beam' strength of the ice. The choice of coefficients is relatively conservative, to ensure that flexural strength is not underestimated.

5. Comprehensive rule equation for angle effects

A comprehensive angle factor, accounting for crushing and flexural failure termed *fa*, is defined as

$$fa = lesser\_of \begin{cases} \left(0.097 - .68 \cdot \left(\frac{x}{L} - .15\right)^2\right) \frac{\alpha}{\sqrt{\beta'}} \\ \\ \frac{1.2 \cdot \sigma_f \cdot h_{ice}^2}{\sin(\beta') \cdot Po^{.36} \cdot \Delta^{.64} \cdot V^{1.28}} \\ \\ 0.6 \end{cases} \qquad (6)$$

As noted above, *fa = 0.6* is the limiting case of the crushing equation. The design ice force is then determined as

$$F_n = fa \cdot Po^{.36} \cdot \Delta_{ship}^{.64} \cdot V_{ship}^{1.28} \qquad (7)$$

The peak force on the bow of a ship depends on the form parameters and on the ice flexure characteristics, as expressed in eqn (6). To examine the behaviour, values for a grid of hull forms (112 cases, see ref [3]) were calculated. The calculations examined the crushing and flexural forces on the bow.

The general tendency is for the crushing force to drop towards the shoulder, while the flexural force tends to rise towards the shoulder. The specific shape of either curve depends, of course, on the hull form. Figure 8 illustrates the nature of the two forces. The upward sloping curve is the flexural force, which increases as the flare angles become more vertical. The downward sloping line is the crushing forces, which reduces due the lower normal velocities. The two curves tend to cross, in which case the point of the crossover (can be anywhere on the bow) defines the maximum force value. The circle represents the peak force (the design force) and its location. This is normally the case on larger and/or lower class ships. On small and/or higher class vessels flexural failure may not matter and the peak force may be right at the stem. In this case, the force is essentially identical to that calculated using the ramming scenario in the longitudinal strength and acceleration calculation, (see ref.[6]).

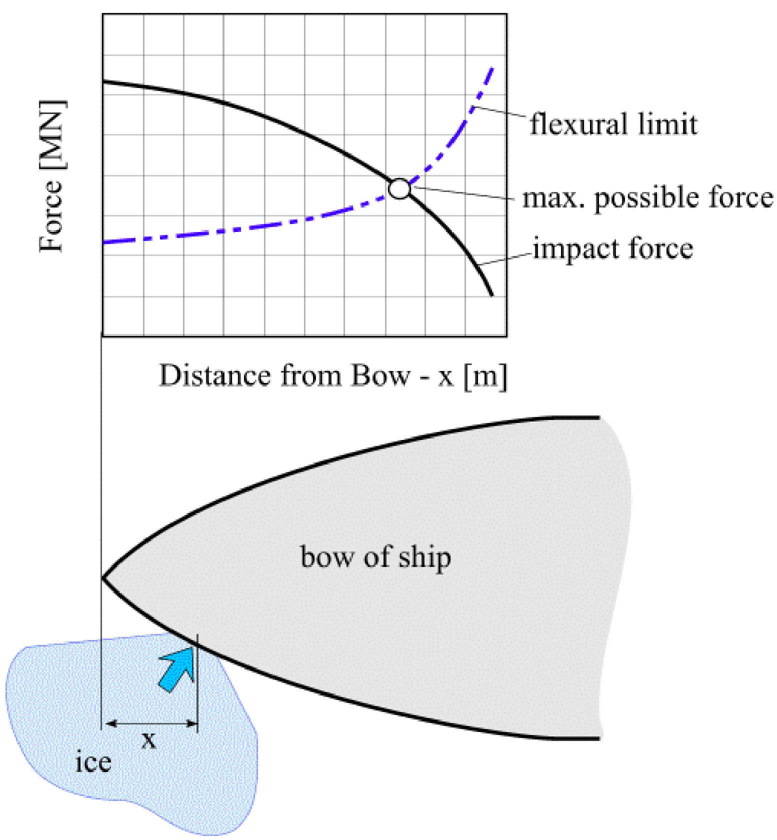

Figure 8. Combination of crushing and flexural forces over the bow of a ship (example).

6. Design load patch

The above discussion has shown how the force can be calculated anywhere on the bow. To continue with the design the load patch (see Figure 9) must be found. The pressure area relationship for ice (see eqns. (a9) and (a10) in Annex A) relates force and nominal contact area. The nominal (overlap) contact area between ship and ice has its shape simplified to an equivalent area rectangular patch. The aspect ratio of this patch is retained, but its area is reduced to account for edge spalling effects observed in ice interactions.. With the force and new patch dimensions, we can find the line load (Q) and the patch pressure (p). Annex A shows the full derivation of these terms. The resulting equations are

$$Q = F_n^{0.611}\, Po^{.389}\, AR^{-0.35} \tag{8}$$

and

$$p = F_n^{0222}\, Po^{.778}\, AR^{0.3} \tag{9}$$

Though we could easily have similar equations for the load length (w) and load height (b) it is more convenient to express w and b in terms of F_n Q and p, as follows,

$$w = F_n / Q \tag{10}$$

$$b = Q/p \tag{11}$$

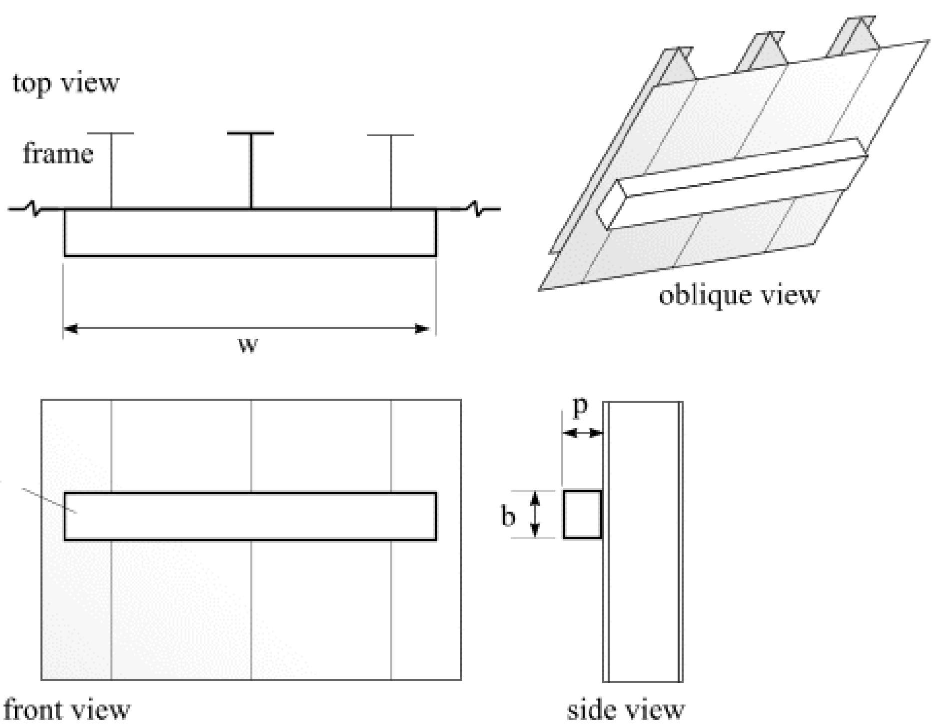

Figure 9. Ice load patch configuration.

7. Rule formulae and method

The sections above show how the ice load is calculated. The rule formulae simplify the equations by using class factors in place of class-dependent physical parameters. The class factors are:

Crushing class factor: $CF_C = Po^{0.36}\ V_{ship}{}^{1.28}$ (12)

Flexural class factor: $CF_F = \sigma_f\ h_{ice}{}^{2}$ (13)

Patch class factor: $CF_D = Po^{.389}$ (14)

Table 1. Class parameters and factors

	Physical Values					Class Factors			
Class	V_{ship} M/s	Po Mpa	h_{ice} M	σ_f Mpa	*D_lim* KT	CF_C	CF_F	CF_D	CF_{DIS}
PC 1	5.70	6.00	7.0	1.40	250	17.7	68.6	2.011	250
PC 2	4.40	4.20	6.0	1.30	210	11.2	46.8	1.750	210
PC 3	3.50	3.20	5.0	1.20	180	7.6	30.0	1.574	180
PC 4	2.75	2.45	4.0	1.10	130	5.0	17.6	1.418	130
PC 5	2.25	2.00	3.0	1.00	70	3.6	9.0	1.310	70
PC 6	2.25	1.40	2.8	0.70	40	3.2	5.5	1.140	40
PC 7	1.75	1.25	2.5	0.65	22	2.2	4.1	1.091	22

Note: D_lim and CF_{DIS} are used in the midbody in lieu of the flexural limit applied in the bow. There is limited theoretical justification for the approach. It is similar to that used in current rules systems and is applied conservatively to larger ships of higher classes. When operating experience with larger ships becomes available this factor should be revisited.

With these class factors, we calculate the bow force as

$$F_n = fa \cdot CF_C \cdot \Delta_{ship}^{.64} \qquad \text{\{bow region\}}$$

$$= fa \cdot CF_C \cdot DF \qquad \text{\{other regions\}} \qquad (15)$$

where

$$fa = lesser\_of \begin{cases} \left(0.097 - .68 \cdot \left(\frac{x}{L} - .15\right)^2\right)\frac{\alpha}{\sqrt{\beta'}} \\ \frac{1.2 \cdot CF_F}{\sin(\beta') \cdot CF_C \cdot \Delta^{.64}} \\ 0.6 \end{cases} \quad \{\text{bow region}\} \qquad (16)$$

$$= 0.36 \qquad \{\text{other hull regions}\}$$

and

$$DF = M_{ship}^{\ 0.64} \qquad if\ M_{ship} < CF_{DIS}$$

$$= CF_{DIS}^{\ 0.64} + 0.1\ (M_{ship} - CF_{DIS}) \qquad if\ M_{ship} < CF_{DIS} \qquad (17)$$

The line load Q is

$$Q = F_n^{\ 0.611} * CF_D / AR^{0.35} \qquad (18)$$

and the patch pressure is

$$p = F_n^{\ 0.222}\ CF_D^{\ 2}\ AR^{0.3} \qquad (19)$$

The aspect ratio is

$$AR = 7.46 \sin(\beta') \quad |\ (\text{not less than } 1.3) \quad \{\text{bow region}\}$$

$$= 3.6 \qquad \{\text{other hull areas}\} \qquad (20)$$

The above formulae are to be calculated at several locations around the bow to ensure that the peak values of *F*, *Q* and *p* are found. Normally, this requires calculations at increments of at least L/20, or at a minimum of five points around the bow area along the deep design ice waterline. This will result in a set of F_i, Q_i, p_i, values (where *i* is the location indicator). In order to create a single design load patch for the whole bow, the largest F_i, Q_i, and p_i, values in the set are selected. They will normally occur at different locations. These maximum values, labeled F_{max}, Q_{max}, p_{max}, are combined to create a conservative load patch, one that has F_{max}, Q_{max}, p_{max} as characteristics. This is achieved by setting the design patch length for the bow as;

$$w_{bow} = F_{max} / Q_{max} \tag{21}$$

$$b_{bow} = Q_{max} / p_{max} \tag{22}$$

In non-bow areas, only a single set of values is calculated, with the normalized values for *fa* and *AR*.

Ice loads are quite peaked within the load patch. To account for this a set of peak pressure factors (*PPF*) is used when using the pressure in design formulae. Figure 10 illustrates how the pressure in the design formula is magnified. The effect of this factor is that smaller structural elements experience larger design pressures. This is another form of pressure-area effect.

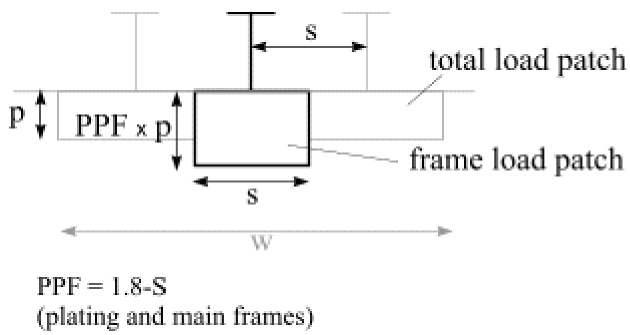

Figure 10. Peak Pressure Factor used to design individual elements.

8. References

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***Annex A** : Derivation of the oblique collision force*

In the following material, the force that results from a ship striking an ice edge is derived. The mechanics are based on the Popov collision but are modified to include a wedge shaped ice edge and a pressure/area ice indentation model.

The force is found by equating the normal kinetic energy with the ice crushing energy,

$$KE_n = E_{crush} \tag{a1}$$

The crushing energy is found by integrating the normal force over the penetration depth,

$$E_{crush} = \int_0^{\delta} F_n(\delta) \cdot d\delta \tag{a2}$$

The normal kinetic energy combines the normal velocity with the effective mass (see Annex B for calculation of the effective mass) at the collision point,

$$KE_n = \frac{1}{2} M_e \cdot V_n^2 \tag{a3}$$

combining these two terms gives

$$\frac{1}{2} M_e \cdot V_n^2 = \int_0^{\delta m} F_n(\delta) \cdot d\delta \tag{a4}$$

where

δ = normal ice penetration

F_n = normal force

M_e = effective mass

= M_{ship}/Co

V_n = normal velocity

=$V_{ship}\ l$

l = direction cosine

The ice penetration geometry together with the pressure-area relationship is the basis of finding the force. The nominal area is found for a penetration δ (see Figure A1).

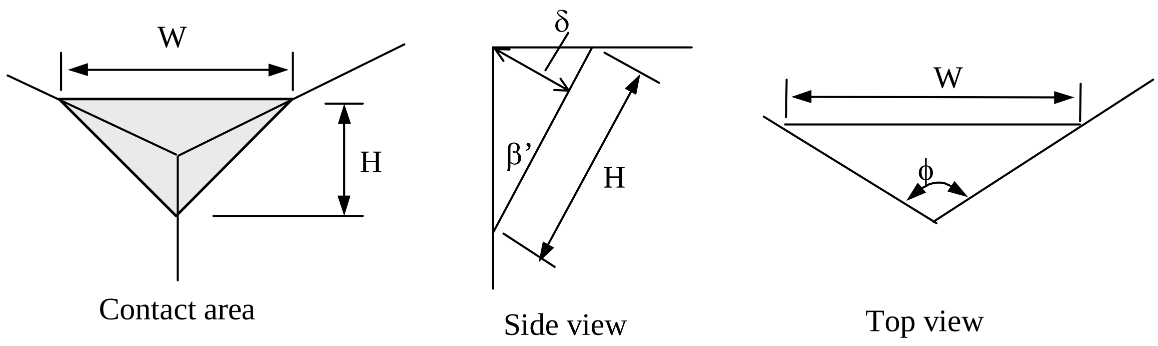

Figure A1 Nominal contact geometry during oblique collision with an ice edge.

The nominal contact area is

$$A = W/2 \times H \tag{a5}$$

The width (W) and height (H) of the nominal contact area can be determined by the normal penetration depth (δ) along with the normal frame angle (β') and the ice edge angle (ϕ),

$$W = 2\ \delta \tan(\phi/2)/\cos(\beta') \tag{a6}$$

$$H = \delta/(\sin(\beta')\cos(\beta')) \tag{a7}$$

Hence the area is

$$A = \delta^2 \tan(\phi/2)/(\cos^2(\beta')\sin(\beta')) \tag{a8}$$

The average pressure is found from the pressure-area relationship,

$$P = Po\ A^{ex} \tag{a9}$$

The normal force is

$$F_n(\delta) = P\ A = Po\ A^{1+ex} \tag{a10}$$

Substituting the expression for area (a8) gives

$$F_n(\delta) = Po\ (\delta^2 \tan(\phi/2)/(\cos^2(\beta')\sin(\beta')))^{1+ex} \tag{a11}$$

$$= Po\ ka^{1+ex}\ \delta^{2+2ex} \tag{a12}$$

where we define the angle factor ka as

$$ka = \tan(\phi/2)/(\cos^2(\beta')\sin(\beta')) \tag{a13}$$

We can now solve the energy balance equation ((a12) into (a4)) to find the maximum penetration,

$$\frac{1}{2} M_e \cdot V_n^2 = Po \cdot ka^{1+ex} \int_0^{\delta m} \delta^{2+2 \cdot ex} \cdot d\delta \qquad \text{(a14)}$$

We can extract the maximum penetration,

$$\delta_m = \left(½\, M_e\, V_n^2\, (3+2ex) / (Po\, ka^{1+ex}) \right)^{1/(3+2ex)} \qquad \text{(a15)}$$

This is substituted into the expression for force, (a12), to give

$$F_n = Po\, ka^{1+ex} \left(½\, M_e\, V_n^2\, (3+2ex) / (Po\, ka^{1+ex}) \right)^{(2+2ex)/(3+2ex)} \qquad \text{(a16)}$$

This can be somewhat simplified to give

$$F_n = Po^{1/(3+2ex)}\, ka^{(1+ex)/(3+2ex)} \left(½\, M_e\, V_n^2\, (3+2ex) \right)^{(2+2ex)/(3+2ex)} \qquad \text{(a17)}$$

Substituting for M_e and V_n, we get

$$\mathrm{F_n = Po^{1/(3+2ex)}\, ka^{(1+ex)/(3+2ex)}\, (l^2 /(2\, Co))^{(2+2ex)/(3+2ex)}\, (M_{ship}\, V_{ship}^2\, (3+2ex))^{(2+2ex)/(3+2ex)}} \qquad \text{(a18)}$$

We can collect all shape related terms (comprising ka and the terms with Co and l) into a single term fa,

$$fa = (3 + 2 \cdot ex)^{\frac{2+2 \cdot ex}{3+2 \cdot ex}} \cdot \left(\frac{\tan(\phi/2)}{\sin(\beta') \cdot \cos^2(\beta')} \right)^{\frac{1+ex}{3+2 \cdot ex}} \cdot \left(\frac{1}{2 \cdot Co} \cdot l^2 \right)^{\frac{2+2 \cdot ex}{3+2 \cdot ex}} \qquad \text{(a19)}$$

With *fa*, we can write the force equation as

$$F_n = fa \cdot Po^{\frac{1}{3+2 \cdot ex}} \cdot V_{ship}^{\frac{4+4ex}{3+2 \cdot ex}} \cdot M_{ship}^{\frac{2+2 \cdot ex}{3+2 \cdot ex}} \qquad \text{(a20)}$$

Which for ex = -0.1 gives

$$F_n = fa\ Po^{0.36}\ V_{ship}^{1.28}\ M_{ship}^{0.64} \qquad \text{(a21)}$$

This value of *fa* collects all form related terms (and constants) into a single factor for crushing. Equation (a21) represents only the crushing force. The flexural failure force must be included in the design force.

The ice load patch is found from F_n. Using (a20) and (a10), we can solve for the nominal contact area,

$$A = \left(\frac{F_n}{Po}\right)^{\frac{1}{1+ex}} \quad \text{(a22)}$$

At this point, we introduce a change in load patch shape from triangular to rectangular. This is done to keep the process manageably simple. We will assume that the load patch is H_{nom} x W_{nom}, with Area A. The aspect ratio AR (W_{nom}/H_{nom}) is

$$AR = 2\ \tan(\phi/2)\ \sin(\beta')$$

$$= 7.46\ \sin(\beta') \qquad \text{[assumes } \phi = 150 \text{ deg]} \quad \text{(a23)}$$

Therefore, we can write

$$A = H_{nom}\ H_{nom}\ AR \quad \text{(a24)}$$

and using (a22)

$$H_{nom} = \left(\frac{F_n}{Po \cdot AR^{1+ex}}\right)^{\frac{1}{2+2ex}} \quad \text{(a25)}$$

$$W_{nom} = \left(\frac{F_n}{Po \cdot AR^{1+ex}}\right)^{\frac{1}{2+2ex}} \cdot AR \quad \text{(a26)}$$

At this point, we introduce a reduction in the size of the load patch (force is unchanged, so design pressure rises correspondingly). This reduction is conservative and is done to account for the typical concentration of force that takes place as ice edges spall off. The rule (or design) patch length w is

$$w = W_{nom}^{wex} = Fn^{wex/(2+2ex)}\ Po^{-wex/(2+2ex)}\ Ar^{wex/2} \quad \text{(a27)}$$

where, with wex = 0.7 and ex = -0.1, we have

$$\text{w} = \text{Fn}^{0.389}\ \text{Po}^{-0.389}\ \text{Ar}^{0.35} \quad \text{(a28)}$$

The design load height is

$$b = \frac{w}{AR} \qquad \text{(a29)}$$

or

$$b = Fn^{0.389}\ Po^{-0.389}\ AR^{-0.65} \qquad \text{(a30)}$$

The nominal and design load patches have the same aspect ratio. The load quantities used in the scantling calculations include the line load,

$$Q = F_n / w \qquad \text{(a31)}$$

and the pressure,

$$p = Q/b \qquad \text{(a32)}$$

We can solve for Q and p by using (a20) and (a22 – a30). The line load becomes

$$Q = \frac{F_n^{1-\frac{wex}{2+2ex}} \cdot Po^{\frac{wex}{2+2ex}}}{AR^{wex/2}} \qquad \text{(a33)}$$

The pressure is

$$p = \frac{F_n^{1-\frac{wex}{1+ex}} \cdot Po^{\frac{wex}{1+ex}}}{AR^{wex-1}} \qquad \text{(a34)}$$

For the rule formula we use *ex* = -0.1, and *wex* = 0.7. This gives;

$$Q = F_n^{0.611}\ Po^{.389}\ AR^{-0.35} \qquad \text{(a35)}$$

and

$$p = F_n^{0.222}\ Po^{.778}\ AR^{0.3} \qquad \text{(a36)}$$

Class Factors

The rules format collects class related parameters into class factors. The following class factors are to be found in the UR:

Crushing class factor $CF_C = Po^{0.36}\ V_{ship}^{\ 1.28}$ (a37)

Flexural class factor $CF_F = \sigma_f\ h_{ice}^{\ 2}$ (a38)

Patch class factor $CF_D = Po^{.389}$ (a39)

With these class factors, we can express the force as

$$F_n = fa\ \ CF_C\ \ M_{ship}{}^{0.64} \tag{a40}$$

The line load and pressure are

$$Q = F_n{}^{0.611}\ CF_D\ \ AR^{-0.35} \tag{a41}$$

$$p = F_n{}^{0.222}\ CF_D{}^2\ AR^{0.3} \tag{a42}$$

respectively.

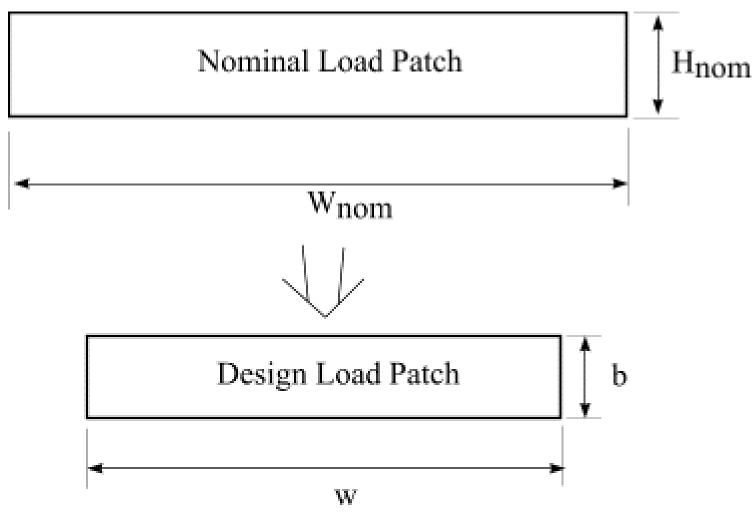

Figure A2. Nominal and design rectangular load patches.

Annex B**: Description of the mass reduction coefficient **Co

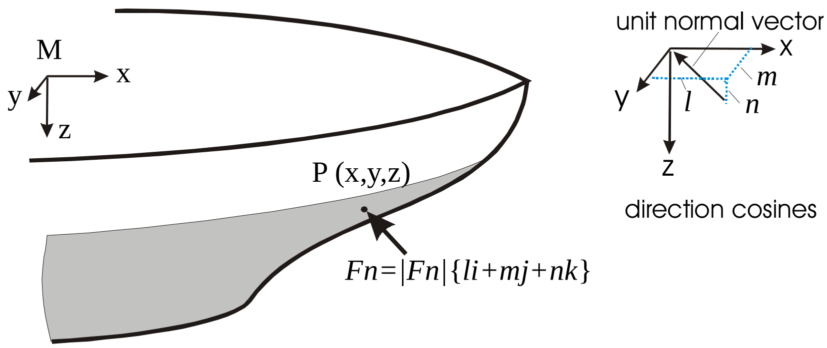

Figure B1. Collision point geometry

A collision taking place at point 'P' (see Figure B1), will result in a normal force *Fn*. Point P will accelerate, and a component of the acceleration will be along the normal vector, with a magnitude a_n. The collision can be modeled as if point P were a single mass (a 1 degree of freedom system) with an equivalent mass *Me* of

$$Me = Fn/a_n \tag{b1}$$

The equivalent mass is a function of the inertial properties (mass, radii of gyration, hull angles and moment arms) of the ship. The equivalent mass is linearly proportional to the mass (displacement) of the vessel, and can be expressed as

$$Me = M_{ship}/Co \tag{b2}$$

where *Co* is the mass reduction coefficient. This approach was first developed by Popov (1972).

The inertial properties of the vessel are as follows,

Hull angles at point P:

α = waterline angle

β = frame angle

β' = normal frame angle

γ = sheer angle

The various angles are related as follows,

$$tan(\beta) = tan(\alpha)\ tan(\gamma) \quad (b3)$$

$$tan(\beta') = tan(\beta)\ tan(\alpha) \quad (b4)$$

Based on these angles, the direction cosines, *l,m,n* are

$$l = sin(\alpha)\ cos(\beta') \quad (b5)$$

$$m = cos(\alpha)\ cos(\beta') \quad (b6)$$

$$n = sin(\beta') \quad (b7)$$

and the moment arms are

$$\lambda 1 = ny\text{-}mz \quad \textit{(roll moment arm)} \quad (b8)$$

$$\mu 1 = lz\text{-}nx \quad \textit{(pitch moment arm)} \quad (b9)$$

$$\eta 1 = mx\text{-}ly \quad \textit{(yaw moment arm)} \quad (b10)$$

The added mass terms are as follows (from Popov),

$$AMx = \textit{added mass factor in surge} = 0 \quad (b11)$$

$$AMy = \textit{added mass factor in sway} = 2\ T/B \quad (b12)$$

$$AMz = \textit{added mass factor in heave} = 2/3\ (B\ Cwp2)/(T(Cb(1+Cwp)) \quad (b13)$$

$$AMrol = \textit{added mass factor in roll} = 0.25 \quad (b14)$$

$$AMpit = \textit{added mass factor in pitch} = B/((T(3\text{-}2Cwp)(3\text{-}Cwp)) \quad (b15)$$

$$AMyaw = \textit{added mass factor in yaw} = 0.3 + 0.05\ L/B \quad (b16)$$

The mass radii of gyration (squared) are

$$rx^2 = Cwp\ B^2/(11.4\ Cm) + H^2/12 \quad \text{(roll)} \quad (b17)$$

$$ry^2 = 0.07\ Cwp\ L^2 \quad \text{(pitch)} \quad (b18)$$

$$rz^2 = L^2/16 \quad \text{(yaw)} \quad (b19)$$

With the above quantities defined, the mass reduction coefficient is

$$Co = l^2/(1+AMx) + m^2/(1+AMy) + n^2/(1+AMz) + \lambda 1^2/(rx^2(1+AMrol) + \mu 1^2/(ry^2\ (1+AMpit)) + \eta 1^2/(rz^2\ (1+AMyaw)) \quad (b20)$$